to the case where the common crack length is large relative to the spacing, contrary to what is suggested by the authors.

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AUTHORS' RESPONSE

The stability arguments presented in the above Discussion do not apply universally to all crack arrays in all situations. As an extreme case, for which the stated crack behaviors are not obtained, one may consider an array such as illustrated in Fig. 1 of the Discussion, but for which cooling of the solid is restricted to the regions immediately surrounding the tips of the shorter cracks, with the longer cracks being heated at their tips. In this instance, the shorter (cooled) cracks would propagate and the longer (heated) cracks would not. Many other such counterexamples can be identified; clearly the stability arguments of the Discussion require very special conditions (e.g. one-dimensional temperature distributions) if they are to hold.

In particular, the conclusions reached in the Discussion do not apply to the arrays of self-driven cracks studied by the authors[1]. There is an essential and important difference between the self-driven cracks and the crack arrays cited in the above Discussion. The analyses of these latter cracks (although they are intended to include the effects of convective fluid flow), consider only one-dimensional temperature fields. The self-driven cracks, on the other hand, experience a temperature distribution which is strongly two-dimensional, with regions of cooling being localized, to varying degrees, near the crack surfaces. While a formal stability analysis of the self-driven crack array has not been carried out, it is clear that the two-dimensionality of the temperature distribution is of fundamental importance, and the results of stability analyses, for cases in which the temperature field is one-dimensional, are not necessarily relevant. In fact, there is good reason to believe that such an array of self-driven cracks would propagate stably (i.e. with all cracks moving at the same speed), as will now be shown.

Consider an infinite array of identical, parallel cracks propagating with a common velocity v away from the surface of a half space (Fig. 1). The crack spacing s is much less than the crack lengths, the half space is initially at a uniform temperature T_0, and a compressive stress σ_0 acts normal to the crack planes. The crack surfaces are assumed to be maintained at the temperature $T_0 - \Delta T$ by fluid flow through the cracks. The mode I stress intensity factor for these cracks can be expressed[2, 3] as

$$K = K_{\sigma_0} + K_{\Delta T}$$

where

$$K_{\sigma_0} = -\sigma_0\sqrt{(s/2)}$$
$$K_{\Delta T} = \Phi(s^*)\sqrt{(s/2)}E\alpha\Delta T/(1-\nu)$$
$$s^* = sv/2c.$$
$$\Phi(s^*) = 1, \qquad \text{for } s^* \leqslant 0.8$$
$$= \sqrt{(0.8/s^*)}, \qquad \text{for } s^* \geqslant 0.8.$$

(The function $\Phi(s^*)$ actually exhibits a smooth transition in the range $0.6 < s^* < 1.0$, but the above representation is generally adequate.) A small value of s^* corresponds to a small crack spacing or a low crack propagation velocity. In this case the temperature field becomes more nearly one-dimensional, with considerable overlapping of the zones of cooling associated with the different cracks. A large value of s^* implies a large spacing or a high velocity and the near-tip cooling zone of any one crack does not interact with those of other cracks.

For $s^* \geqslant 0.8$ these results give

$$K = \sqrt{(0.8c/v)}E\alpha\Delta T/(1-\nu) - \sigma_0\sqrt{(s/2)}.$$

Thus, for steady-state propagation of widely spaced (or fast) cracks, the (positive) contribution of cooling to the stress intensity factor is independent of the spacing, while the (negative) contribution of the confining stress increases as the square root of the spacing. The propagation velocity, obtained by setting K equal to the critical value K_c, is

$$v = 0.8c[E\alpha\Delta T/(1-\nu)]^2/[K_c + \sigma_0\sqrt{(s/2)}]^2.$$

If a crack were to become longer than neighboring cracks, it would experience stresses similar to those of a crack in an array with a larger spacing. The propagation velocity of such a crack would be lower than the surrounding (more closely spaced) cracks, allowing the rest of the array to overtake the longer crack.

If a crack were to fall behind neighboring cracks, it would experience the competing effects of (1) an increased stress intensity factor as a result of the longer cracks shielding it from the compressive confining stress, and (2) a decreased stress intensity factor resulting from the tensile thermal stresses ahead of the crack array being replaced by the stress-free surfaces of the adjacent longer cracks. In the limit of very large s^*, however, there are no thermal stresses ahead of the crack array[3] so the shorter crack experiences only the

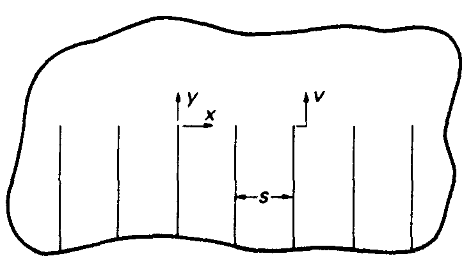

Fig. 1. An infinite array of identical, parallel cracks propagating with velocity v away from the surface of a half space. The crack spacing s is much less than the crack lengths. The surface of the half space is parallel to the x–z plane. The solid has a modulus of elasticity E, Poisson's ratio ν, thermal diffusivity c, and linear coefficient of thermal expansion α.

shielding effect, resulting in an increased stress intensity factor. A shorter crack would therefore develop a greater propagation speed, allowing it to catch up to the longer cracks.

Thus, an examination of both longer and shorter cracks indicates that the array of self-driven cracks, for large enough values of s^*, is a stable configuration: cracks which deviate from the common length and speed experience stresses which act to restore uniformity to the array.

While the above considerations should be complemented by a complete stability analysis of the crack array, they demonstrate the importance of taking the two-dimensionality of the problem into consideration. They also demonstrate that the self-driven crack array can be expected to be stable in certain cases. It can be shown[3] that these cases include situations of practical interest. The detailed stability analysis is an appropriate topic for further study.

The ability of an array of cracks to develop a self-driven character depends, of course, on sufficient flow of fluid through the cracks to provide the required cooling. Since characteristic crack opening widths are $w = s\alpha\Delta T$, and since the fluid flow rate through the cracks is proportional to w^3, closely-spaced cracks will not have sufficient fluid flow to be self-driven. However, arrays of closely-spaced cracks, propagating under the influence of a one-dimensional conduction-controlled temperature field, will increase their spacing as they propagate, allowing an increased fluid flow rate and eventually admitting the possibility of self-driven behavior[3]. The Discussion seems relevant only in the early regime preceding this mature phase of extensive propagation.

As well the experimental results described in the Discussion above, for cracks growing from the cooled edge of a glass plate, are not pertinent to self-driven crack arrays, since the temperature distributions used are one-dimensional, with no fluid flow to cool the crack surfaces. The results described are similar to those reported by the authors for experiments in which large ($44 \times 22 \times 11$ in) blocks of ice had a portion of one face cooled and a quasi-one-dimensional temperature distribution produced cracking[4]. It is noted that interpretation of the results from the experiments described in the Discussion is further complicated by the fact that the glass plates are in a state of plane stress everywhere except near the crack tips where the variation in stresses is so intense that a plane strain condition mat apply.

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